Ohio County Populations and Areas

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Teachers are always seeking situations in which mathematics can be connected to real world data. The counties of a given state can be used as a setting for these connections.

Table I is a listing of the population, area (square miles), and "density" (population ÷ area) for each of the 88 counties of Ohio. The population and area data were found in a recent world almanac; the density was computed from the other data.

Table I

County	Population	Area (square miles)	Density	County	Population	Area (square miles)	Density
Adams	25,371	586	43.3	Licking	128,300	686	187.0
Allen	109,755	405	271.0	Logan	42,310	458	92.4
Ashland	47,507	424	112.0	Lorain	271,126	495	547.7
Ashtabula	99,821	703	142.0	Lucas	462,361	341	1,355.9
Athens	59,549	508	117.2	Madison	37,068	467	79.4
Auglaize	44,585	398	112.0	Mahoning	264,806	417	635.0
Belmont	71,074	537	132.4	Marion	64,274	403	159.5
Brown	34,966	493	70.9	Medina	122,354	422	289.9
Butler	291,479	470	620.2	Meigs	22,987	432	53.2
Carroll	26,521	393	67.5	Mercer	39,443	457	86.3
Champaign	36,019	429	84.0	Miami	93,182	410	227.3
Clark	147,548	398	370.7	Monroe	15,497	457	33.9
Clermont	150,167	456	329.3	Montgomery	573,809	458	1,252.9
Clinton	35,417	410	86.4	Morgan	14,194	420	33.8
Columbiana	108,276	534	202.8	Morrow	27,749	406	68.3
Coshocton	35,427	566	62.6	Muskingum	82,068	654	125.5
Crawford	47,870	403	118.8	Noble	11,336	399	28.4
Cuyahoga	1,412,140	459	3,076.6	Ottawa	40,029	253	158.2

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Darke	53,619	600	89.4	Paulding	20,488	419	48.9
Defiance	39,350	414	95.0	Perry	31,557	412	76.6
Delaware	66,929	443	151.1	Pickaway	48,244	503	95.9
Erie	76,779	264	290.8	Pike	24,249	443	54.7
Fairfield	103,472	506	204.5	Portage	142,585	493	289.2
Fayette	27,466	405	67.8	Preble	40,113	426	94.2
Franklin	961,437	543	1,770.6	Putnam	33,819	484	69.9
Fulton	38,498	407	94.6	Richland	126,137	497	253.8
Gallia	30,954	471	65.7	Ross	69,330	692	100.2
Geauga	81,129	408	198.8	Sandusky	61,963	409	151.5
Greene	136,731	416	328.7	Scioto	80,327	613	131.0
Guernsey	39,024	522	74.8	Seneca	59,733	553	108.0
Hamilton	866,228	412	2,102.5	Shelby	44,915	409	109.8
Hancock	65,536	532	123.2	Stark	367,585	574	640.4
Hardin	31,111	471	66.1	Summit	514,990	412	1,250.0
Harrison	16,085	400	40.2	Trumbull	227,813	612	372.2
Henry	29,108	415	70.1	Tuscarawas	84,090	570	147.5
Highland	35,728	553	64.6	Union	31,969	437	73.2
Hocking	25,533	423	60.4	Van Wert	30,464	410	74.3
Holmes	32,849	424	77.5	Vinton	11,098	414	26.8
Huron	56,240	494	113.8	Warren	113,927	403	282.7
Jackson	30,230	420	72.0	Washington	62,254	640	97.3
Jefferson	80,298	410	195.8	Wayne	101,461	557	182.2
Knox	47,473	529	89.7	Williams	36,956	422	87.6
Lake	215,499	231	932.9	Wood	113,269	619	183.0
Lawrence	61,834	457	135.3	Wyandot	22,254	406	54.8
				Totals	**10,847,115**	**41,006**	

Questions for your students based upon the data of Table I.

A. Rank order the 88 counties by population.

1. Find the mean and median of the 88 county populations. What does the inequality of these two measures tell about the skewness of the data?
   (Mean = 123,262.7; median = 54,929.5, skewed to the right.)
2. What is the smallest number of counties needed to account for one-half of Ohio's population of 10,847,115? (Eight counties: Cuyahoga, Franklin, Hamilton, Montgomery, Summit, Lucas, Stark, and Butler.)

3. Fill in the blank with the largest possible number: Cuyahoga county has a larger population than the combined total of the _____ least populated counties. (The sum of the populations of the 43 least populated counties is 1,383,831; this is just less than Cuyahoga county's population of 1,412,140.)
4. Redo problem 3 for the top two counties - Cuyahoga and Franklin. (The least populated 57 counties are needed.)

B. Rank order the 88 counties by area.
1. Find the mean and median of the 88 county areas. What does the inequality of the two measures say about the skewness of the data?
(Mean = 466.0; median = 440.0; little apparent skewing.)
2. What is the smallest number of counties needed to account for one-half of Ohio's total area of 41,006 square miles? (38 counties are needed - from Ashtabula to Logan.)
3. Fill in the blank with the largest possible number: Ashtabula has a larger area than the combined areas of the _____ smallest counties. (The sum of the areas of the 2 smallest counties is 484 square miles, just less than the area of Ashtabula, which is 703 square miles.)
4. Redo problem 3 for the top two counties in area; these are Ashtabula and Paulding. (4). The answers to questions 3 and 4 emphasize that the county areas generally occur in a narrow band.

C. Rank order the 88 counties by density.
1. Find the total population and area of the three most densely populated counties. (Cuyahoga, Hamilton, and Franklin have a total population of 3,239,805 and a total area of 1,414 square miles.)
2. What is the density of the three-county group? (3,239,805 ÷ 1,414 = 2,291.2)
3. If the entire state of Ohio were as densely populated as this three-county group, what would be the state's population? (x = 93,954,345. This is approximately the combined populations of the six New England states (Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, and Vermont), plus Illinois, Indiana, Kentucky, Michigan, Missouri, New Jersey, New York, Pennsylvania, West Virginia, and Wisconsin.)
4. If the entire state of Ohio were as densely populated as its three least densely populated counties, what would be the state's population? (Total population = 36,628, total area = 1,233 square miles; $\frac{36,628}{1,233} = \frac{x}{41,006}$; x = 1,218,141. This is just about the population of the state of Maine.)
5. If the United States were as densely populated as Ohio's three most densely populated counties, what would be the population of the United States? (Since the total area of the United States is 3,787,319 square miles, we must solve the following equation: $\frac{3,239,805}{1,414} = \frac{x}{3,787,319}$; x = 8,677,630,000. This is approximately 50% larger than the population of the entire world.)
6. If the United States were as densely populated as Ohio's three least densely populated counties, what would be the population of the United States? ($\frac{36,628}{1,233} = \frac{x}{3,787,319}$; x = 112,507,000. This is close to half the actual population of the United States.)

Challenges for the reader and his/her students:

A. Perform the same analysis for:
   1. other states.
   2. the United States and its 50 states.
   3. other countries and their subdivisions.

B. Use boxplots in looking for data trends.

C. On an outline of Ohio and its counties, shade the most densely populated counties. Compare this map to ones on which the counties with the most economic activity and pollution are shaded.

D. Find other situations in which data can be manipulated and characterized.

Answer to question one from the article "Two Problems for 2002."

1. 2002--It's Sum Year.

a.) Each proper odd divisor of 2002 leads to one of the possible summations of consecutive positive integers.

2002 = 7 x 286 = 283 + 284 + 285 + 286 + 287 + 288 + 289,
2002 = 11 x 182 = 177 + 178 + ... + 182 + ... + 186 + 187,
2002 = 13 x 154 = 148 + 149 + ... + 154 + ... + 159 + 160,
2002 = 1001 x 2 = 499 + (500 + 501) + 502,
2002 = 143 x 14 = 58 + 59 + ... + (71 + 72) + ... + 84 + 85,
2002 = 91 x 22 = 24 + 25 + ... + (45 + 46) + ... + 66 + 67,
2002 = 77 x 26 = 13 + 14 + ... + (38 + 39) + ... + 63 + 64.

b.) Each proper odd divisor of 2002 leads to one of the possible summations of consecutive positive even integers.

2002 = 1001 x 2 = 1000 + 1002,
2002 = 7 x 286 = 280 + 282 + 284 + 286 + 288 + 290 + 292,
2002 = 11 x 182 = 172 + 174 + ... + 182 + ... + 190 + 192,
2002 = 13 x 154 = 142 + 144 + ... + 154 + ... + 164 + 166,
2002 = 77 x 26 = 52 + 54 + ... + (76 + 78) + ... + 100 + 102,
2002 = 91 x 22 = 70 + 72 + ... + (90 + 92) + ... + 110 + 112,
2002 = 143 x 14 = 130 + 132 + ... + (142 + 144) + ... + 154 + 156.

c.) The sum of an odd number of consecutive odd numbers is always odd. The sum of an even number of consecutive odd numbers is always a multiple of 4. Since 2002 is even and not a multiple of 4, 2002 cannot be written as a sum of consecutive odd integers.

Reference

Prielipp and Kuenzi, "Sums of Consecutive Positive Integers", Mathematics Teacher, January, 1975.